Tipler Physics Problem 2-61

A passenger is running at her maximum velocity of 8 m/s to catch a train. When she is a distance d from the nearest entry to the train, the train starts from rest with a constant acceleration a = 1.0 m/s^2 away from her.

(a) If d = 30 m and the passenger keeps running, will she be able to jump onto the train? (Assume she is not Michael Jordan's sister … she probably doesn't have to run for trains!)

(b) Carefully sketch the position function x(t) for the train, choosing x = 0 at t = 0. On the same graph sketch x(t) for the passenger for various initial separation distances d, including d = 30m and the critical separation distance d_c such that she just catches the train. Use Mathematica if you can.

(c) For the critical separation distance, what is the speed of the train when the passenger catches it? What is the train's average speed for the time interval from t = 0 until she catches it?

Using the kinematics equations for the passenger & train gives

In[2]:=
```
xp = vp * tcatch;
xt = do + vit tcatch + 1/2 at tcatch^2;
```

To find t_{catch} equate x_p to x_{train} and get a quadratic:

In[4]:=
```
Solve[vp * tcatch == do + vit tcatch + 1/2 atrain tcatch^2, tcatch]
```

Out[4]=
$$\left\{\left\{t_{catch} \to \frac{-2\,v_{it} - \sqrt{-8\,a_{train}\,d_o + (2\,v_{it} - 2\,v_p)^2} + 2\,v_p}{2\,a_{train}}\right\},\ \left\{t_{catch} \to \frac{-2\,v_{it} + \sqrt{-8\,a_{train}\,d_o + (2\,v_{it} - 2\,v_p)^2} + 2\,v_p}{2\,a_{train}}\right\}\right\}$$

The numerical values (in mks) give the two values of the catch time in seconds to make sure they're equal and find out their values.

In[5]:=
```
do = 30;
vp = 8;
vit = 0;
at = 1;
NSolve[vp * tcatch == do + vit tcatch + 1/2 at tcatch^2, tcatch]
```

Out[9]=
$$\{\{t_{catch} \to 6.\}, \{t_{catch} \to 10.\}\}$$

To plot the motions of the passenger and train, redefine them in terms of t, not t_{catch} and evaluate each at both possibilites of t_{catch}.

In[10]:=
$$x_{pass} = v_p * t;$$
$$x_{train} = d_o + v_{it}\, t + \frac{1}{2}\, a_t\, t^2;$$

In[12]:=
$$t_{low} = 6;$$
$$t_{high} = 10;$$
$$x_{passlow} = v_p * t_{low}$$
$$x_{trainlow} = d_o + v_{it}\, t_{low} + \frac{1}{2}\, a_t\, t_{low}^2$$
$$x_{passhigh} = v_p * t_{high}$$
$$x_{trainhigh} = d_o + v_{it}\, t_{high} + \frac{1}{2}\, a_t\, t_{high}^2$$

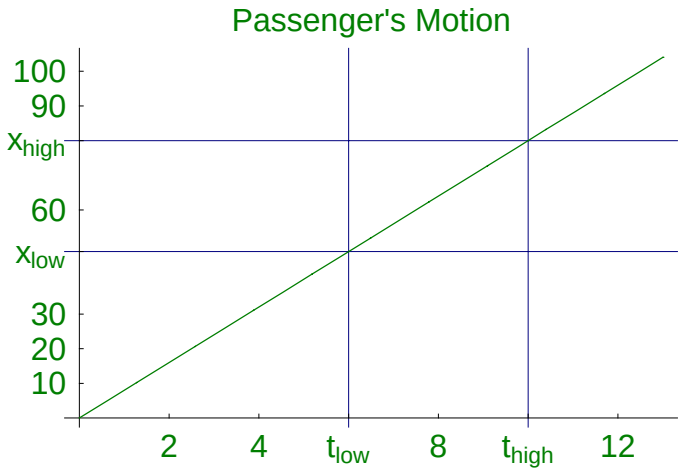
Out[14]= 48

Out[15]= 48

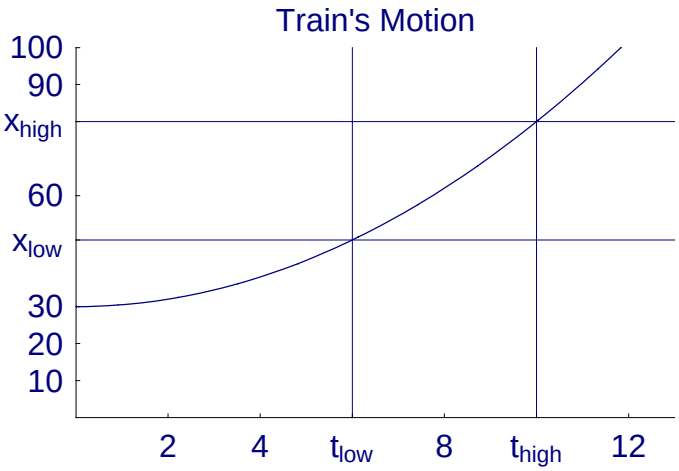
Out[16]= 80

Out[17]= 80

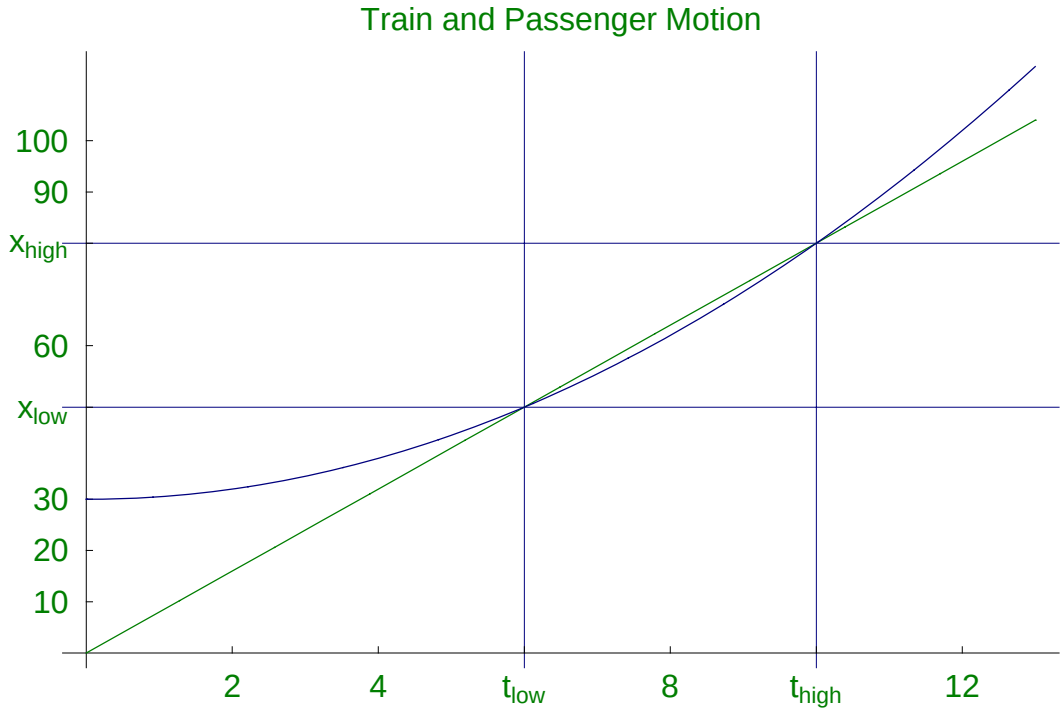
In[17]:=
```
pPassenger = Plot[x_pass, {t, 0, 13}, TextStyle →
   {FontFamily → Helvetica, FontSize → 12, FontColor -> RGBColor[0, 0.5, 0]},
  GridLines → {{6, 10}, {x_passlow, x_passhigh}},
  Ticks → {{2, 4, {6, "t_low"}, 8, {10, "t_high"}, 12},
    {10, 20, 30, {48, "x_low"}, 60, {80, "x_high"}, 90, 100}},
  PlotPoints → 500, PlotStyle → {RGBColor[0, 0.5, 0]}, PlotRange → All,
  PlotLabel → "Passenger's Motion"]
```



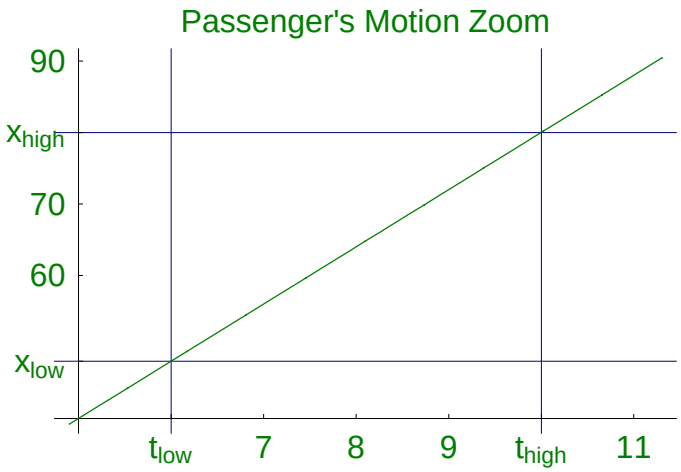
In[18]:=
```
pTrain = Plot[xtrain, {t, 0, 13}, TextStyle →
   {FontFamily → Helvetica, FontSize → 12, FontColor -> RGBColor[0, 0, 0.5]},
  GridLines → {{6, 10}, {xpasslow, xpasshigh}},
  Ticks → {{0, 2, 4, {6, "tlow"}, 8, {10, "thigh"}, 12},
    {0, 10, 20, 30, {48, "xlow"}, 60, {80, "xhigh"}, 90, 100}},
  PlotPoints → 500, PlotStyle → {RGBColor[0, 0, 0.5]},
  PlotRange → {{0, 13}, {0, 100}},
  PlotLabel → "Train's Motion"]
```



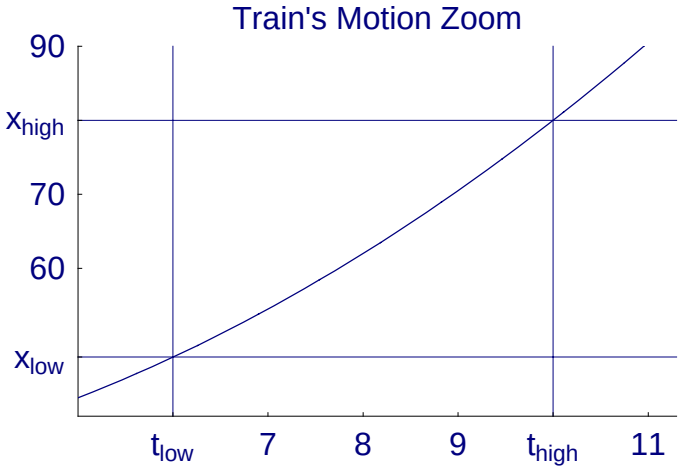
In[19]:=
```
Show[pPassenger, pTrain, PlotLabel → "Train and Passenger Motion"]
```



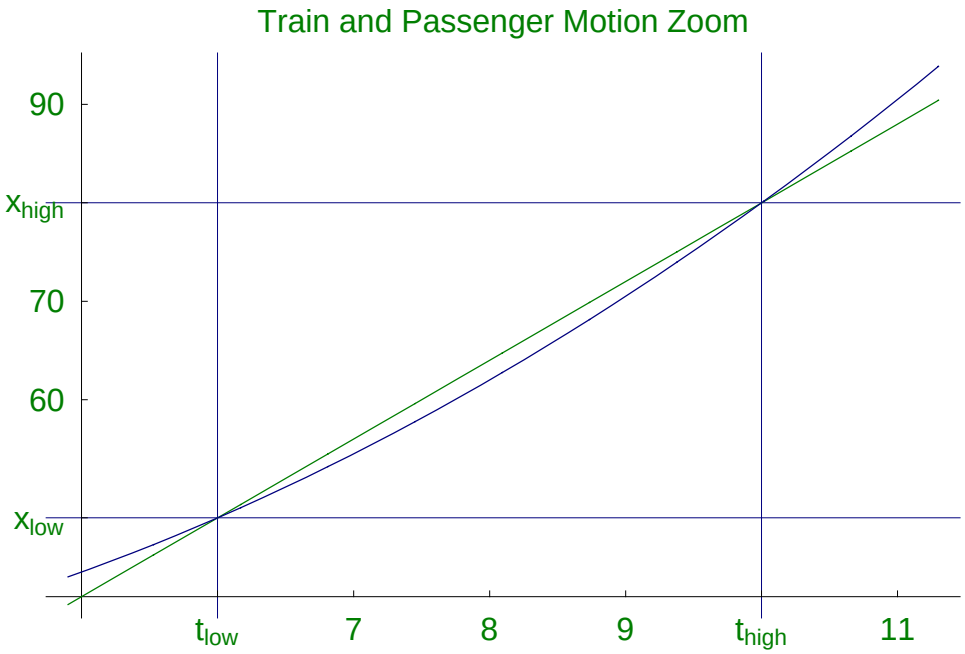
In[20]:=
```
pPassengerZoom = Plot[xpass, {t, 4.9, 11.3}, TextStyle →
   {FontFamily → Helvetica, FontSize → 12, FontColor -> RGBColor[0, 0.5, 0]},
  GridLines → {{6, 10}, {xpasslow, xpasshigh}},
  Ticks → {{5, {6, "tlow"}, 7, 8, 9, {10, "thigh"}, 11},
    {40, {48, "xlow"}, 60, 70, {80, "xhigh"}, 90}},
  PlotPoints → 500, PlotStyle → {RGBColor[0, 0.5, 0]}, PlotRange → All,
  PlotLabel → "Passenger's Motion Zoom"]
```



In[21]:=
```
pTrainZoom = Plot[xtrain, {t, 4.9, 11.3}, TextStyle →
   {FontFamily → Helvetica, FontSize → 12, FontColor -> RGBColor[0, 0, 0.5]},
  GridLines → {{6, 10}, {xpasslow, xpasshigh}},
  Ticks → {{5, {6, "tlow"}, 7, 8, 9, {10, "thigh"}, 11},
    {40, {48, "xlow"}, 60, 70, {80, "xhigh"}, 90, 100}},
  PlotPoints → 500, PlotStyle → {RGBColor[0, 0, 0.5]},
  PlotRange → {{5, 11.3}, {40, 90}},
  PlotLabel → "Train's Motion Zoom"]
```



In[22]:=
```
Show[pPassengerZoom, pTrainZoom,
 PlotLabel → "Train and Passenger Motion Zoom"]
```



To find the critical distance, where the passenger just catches the train, set the radical in the quadratic to zero and find d_c.

In[18]:=
```
QuadRad = Sqrt[vp^2 - 2 at dc]
NSolve[QuadRad == 0, dc]
```

Out[18]= $\sqrt{64 - 2\, d_c}$

Out[19]= $\{\{d_c \to 32.\}\}$

In[20]:=
```
dc = 32;
tcrit = Sqrt[(2 * dc)/at]
```

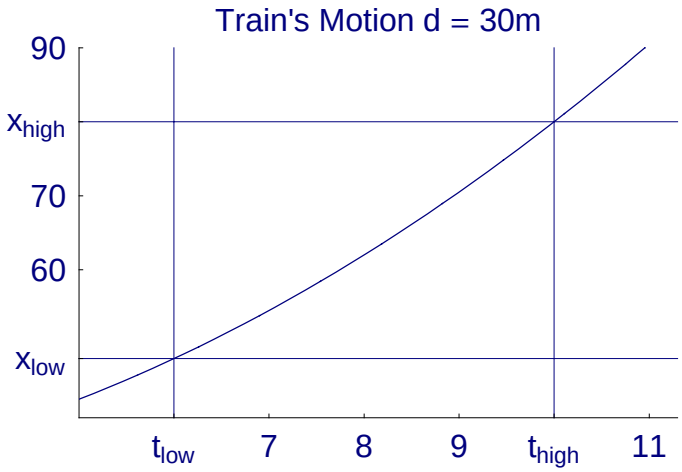
Out[21]= 8

Plot the Train's motion for different initial disances and superimpose it on the passenger's motion

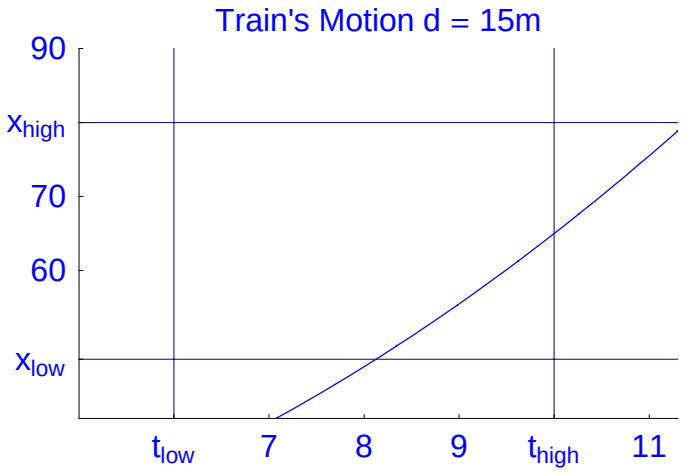
In[22]:=
```
dist + vit t + 1/2 at t^2
vit = 0;
at = 1;
```

Out[22]= $\text{dist} + \frac{t^2}{2}$

In[28]:=
```
pTrain30 = Plot[30 + t^2/2, {t, 4.9, 11.3}, TextStyle →
  {FontFamily → Helvetica, FontSize → 12, FontColor -> RGBColor[0, 0, 0.5]},
 GridLines → {{6, 10}, {xpasslow, xpasshigh}},
 Ticks → {{5, {6, "tlow"}, 7, 8, 9, {10, "thigh"}, 11}, {40, {48, "xlow"},
    60, 70, {80, "xhigh"}, 90, 100}}, PlotRange → {{5, 11.3}, {40, 90}},
 PlotPoints → 500, PlotStyle → {RGBColor[0, 0, 0.5]},
 PlotLabel → "Train's Motion d = 30m"]
```

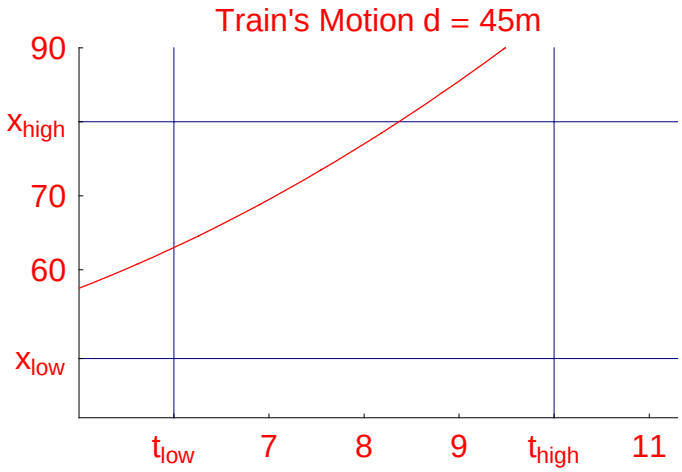


In[29]:=
```
pTrain15 = Plot[15 + t^2/2, {t, 4.9, 11.3}, TextStyle →
  {FontFamily → Helvetica, FontSize → 12, FontColor -> RGBColor[0, 0, 1]},
 GridLines → {{6, 10}, {xpasslow, xpasshigh}},
 Ticks → {{5, {6, "tlow"}, 7, 8, 9, {10, "thigh"}, 11}, {40, {48, "xlow"},
    60, 70, {80, "xhigh"}, 90, 100}}, PlotRange → {{5, 11.3}, {40, 90}},
 PlotPoints → 500, PlotStyle → {RGBColor[0, 0, 1]},
 PlotLabel → "Train's Motion d = 15m"]
```



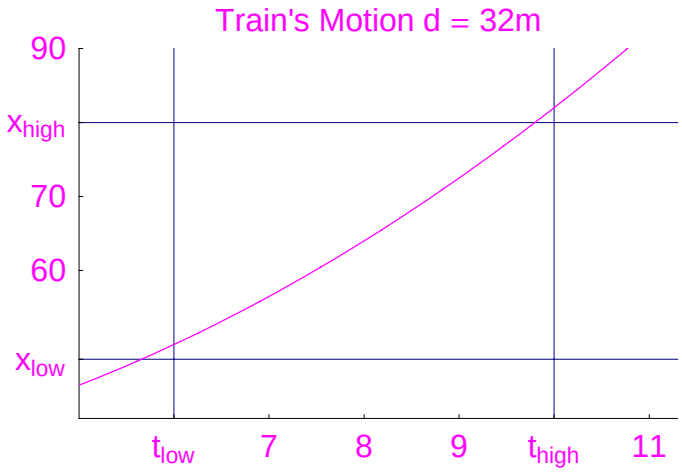
In[30]:=

```
pTrain45 = Plot[45 + t^2/2, {t, 4.9, 11.3}, TextStyle →
  {FontFamily → Helvetica, FontSize → 12, FontColor -> RGBColor[1, 0, 0]},
 GridLines → {{6, 10}, {xpasslow, xpasshigh}},
 Ticks → {{5, {6, "tlow"}, 7, 8, 9, {10, "thigh"}, 11}, {40, {48, "xlow"},
    60, 70, {80, "xhigh"}, 90, 100}}, PlotRange → {{5, 11.3}, {40, 90}},
 PlotPoints → 500, PlotStyle → {RGBColor[1, 0, 0]},
 PlotLabel → "Train's Motion d = 45m"]
```

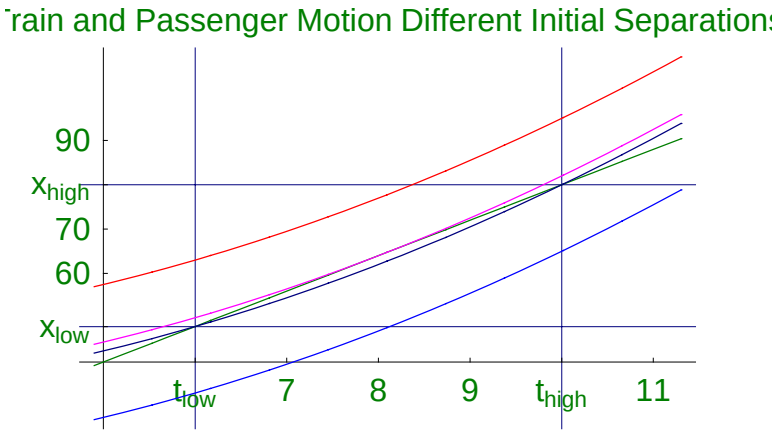


In[31]:=

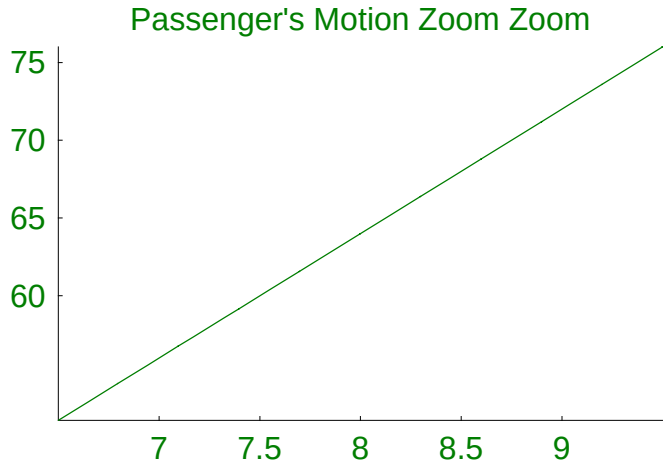
```
pTrain32 = Plot[32 + t^2/2, {t, 4.9, 11.3}, TextStyle →
  {FontFamily → Helvetica, FontSize → 12, FontColor -> RGBColor[1, 0, 1]},
 GridLines → {{6, 10}, {xpasslow, xpasshigh}},
 Ticks → {{5, {6, "tlow"}, 7, 8, 9, {10, "thigh"}, 11}, {40, {48, "xlow"},
    60, 70, {80, "xhigh"}, 90, 100}}, PlotRange → {{5, 11.3}, {40, 90}},
 PlotPoints → 500, PlotStyle → {RGBColor[1, 0, 1]},
 PlotLabel → "Train's Motion d = 32m"]
```



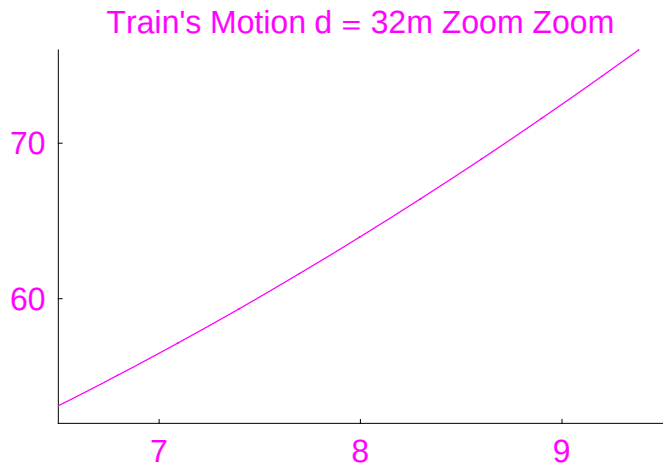
In[34]:=
```
Show[pPassengerZoom, pTrain30, pTrain45, pTrain32, pTrain15,
 PlotLabel → "Train and Passenger Motion Different Initial Separations"]
```



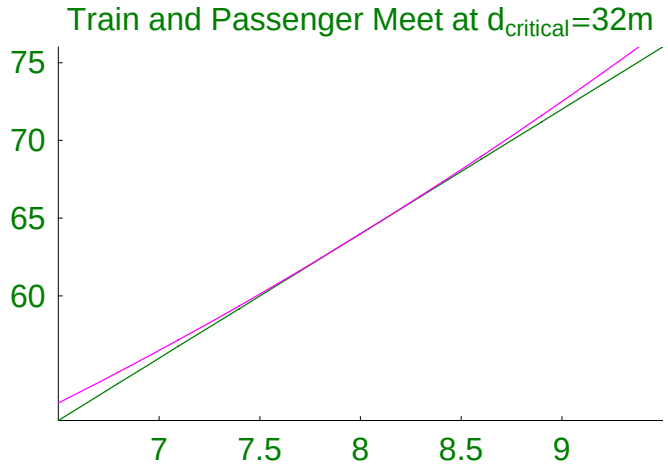
In[53]:=
```
pPassengerZoomZoom = Plot[x_pass, {t, 6.5, 9.5}, TextStyle →
  {FontFamily → Helvetica, FontSize → 12, FontColor -> RGBColor[0, 0.5, 0]},
 Ticks → {{6.5, 7, 7.5, 8, 8.5, 9}, {60, 65, 70, 75, 80, 85}},
 AxesOrigin → {6.5, 52},
 PlotPoints → 500, PlotStyle → {RGBColor[0, 0.5, 0]},
 PlotRange → {{6.5, 9.5}, {52, 76}},
 PlotLabel → "Passenger's Motion Zoom Zoom"]
```



In[60]:=
```
pTrain32ZoomZoom = Plot[32 + t^2/2, {t, 6.5, 9.5}, TextStyle →
  {FontFamily → Helvetica, FontSize → 12, FontColor -> RGBColor[1, 0, 1]},
 Ticks → {{6, 7, 8, 9}, {60, 70}}, AxesOrigin → {6.5, 52},
 PlotRange → {{6.5, 9.5}, {52, 76}},
 PlotPoints → 500, PlotStyle → {RGBColor[1, 0, 1]},
 PlotLabel → "Train's Motion d = 32m Zoom Zoom"]
```



In[62]:=
```
Show[pPassengerZoomZoom, pTrain32ZoomZoom,
 PlotLabel → "Train and Passenger Meet at d_critical=32m"]
```



Find the velocity of the train when the passenger catches it at the critical distance and the average speed of the train from the start to this point

In[29]:=
```
vtrain = vit + at tcrit
vaverage = (vtrain + vit)/2
```

Out[29]= 8

Out[30]= 4